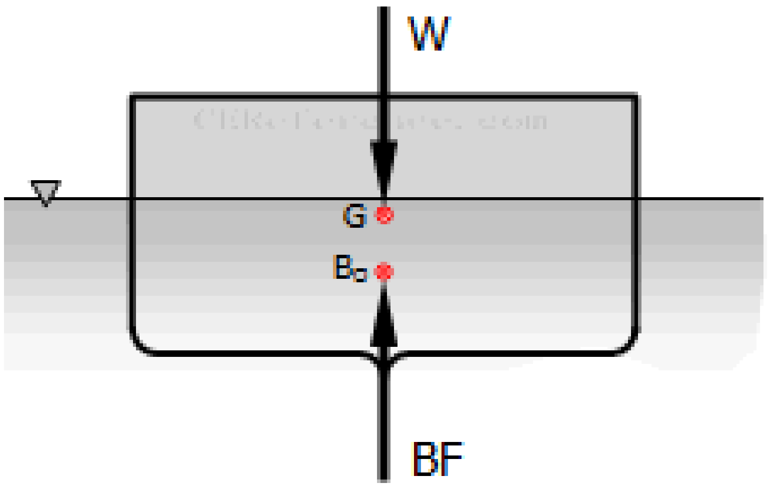
HYDRO STATICS

STABILITY OF FLOATING BODIES

Fluid Action on Surfaces

Stability of Floating Bodies

Any floating body is subjected by two opposing vertical forces. One is the body's weight W which is downward, and the other is the buoyant force BF which is upward. The weight is acting at the center of gravity G and the buoyant force is acting at the center of buoyancy B_O. W and BF are always equal and if these forces are collinear, the body will be in upright position as shown.

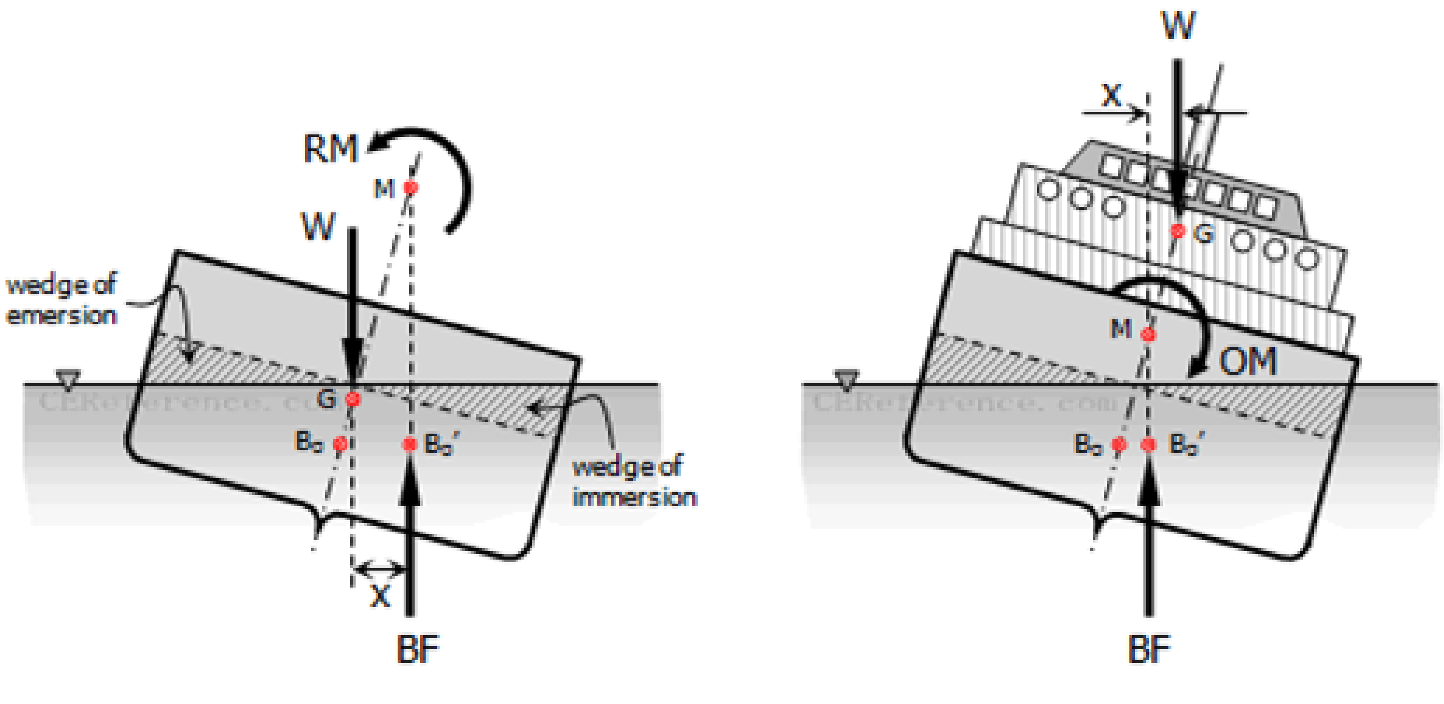


Upright Position

Fluid Action on Surfaces

Stability of Floating Bodies

The body may tilt from many causes like wind or wave action causing the center of buoyancy to shift to a new position B_O' as shown below.



M is above G: Stable Position

M is below G: Unstable Position

Fluid Action on Surfaces

Stability of Floating Bodies

Point M is the intersection of the axis of the body and the line of action of the buoyant force, it is called metacenter. If M is above G, BF and W will produce a righting moment RM which causes the body to return to its neutral position, thus the body is stable. If M is below G, the body becomes unstable because of the overturning moment OM made by W and BF. If M coincides with G, the body is said to be just stable which simply means critical. The value of righting moment or overturning moment is given by:

$$RM \text{ or } OM = Wx = W\,(MG \sin\theta)$$

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The distance MG is called metacentric height.

$$\text{Metacentric height, } MG = MB_O \pm GB_O$$

Use (-) if G is above BO and (+) if G is below BO. Note that M is always above BO.

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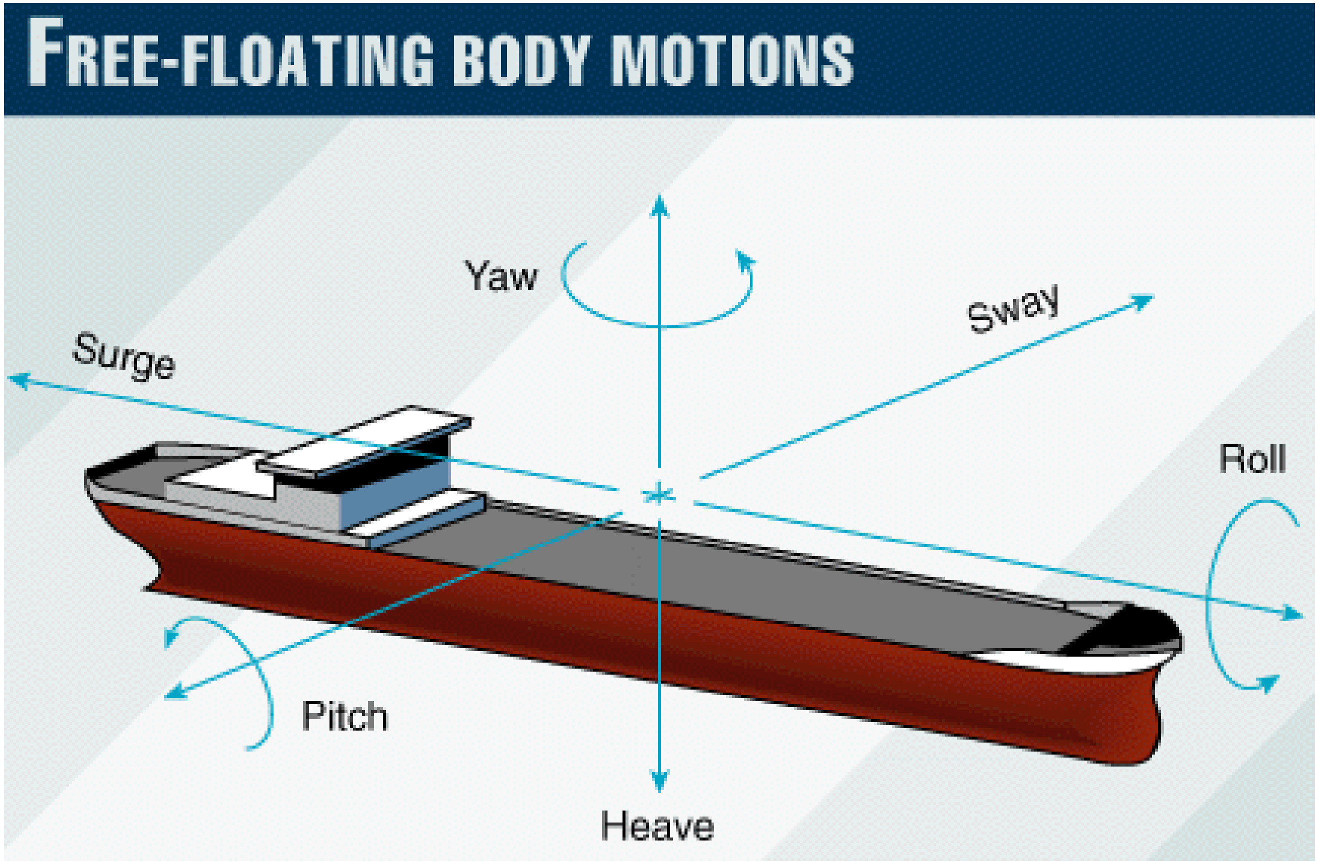
Stability of Floating Bodies

Value of MB_O



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Stability of Floating Bodies

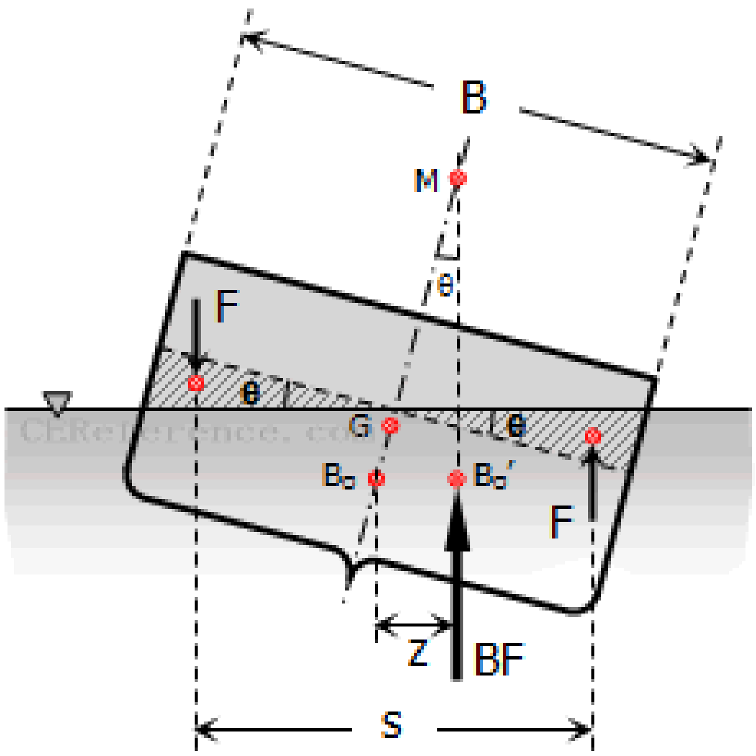
Value of MB_O

Assume that the body is rectangular at the top view and measures B by L at the waterline when in upright position. The moment due to the shifting of the buoyant force is equal to the moment due to shifting of wedge.

$$BF\,z = F\,s$$

$$\gamma V_D\,(MB_O \sin\theta) = (\gamma v)s$$

$$V_D MB_O \sin\theta = vs$$



The length is L perpendicular to the drawing

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Stability of Floating Bodies

Value of MB_O

$$MB_O = \frac{vs}{V_D \sin\theta}$$

$$MB_O = \frac{[\,\frac{1}{2}(\frac{1}{2}B)(\frac{1}{2}B\tan\theta)L\,](\frac{2}{3}B)}{V_D \sin\theta}$$

$$MB_O = \frac{\frac{1}{12}LB^3\tan\theta}{V_D \sin\theta}$$

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Stability of Floating Bodies

Value of MB_O

For small value of θ, $\tan\theta \equiv \sin\theta$ and note that $1/12\ LB^3 = I$, thus,

$$MB_O = \frac{I\sin\theta}{V_D\sin\theta}$$

$$MB_O = \frac{I}{V_D}$$

The formula above can be applied to any section.

Fluid Action on Surfaces

Where

W = weight of the body

BF = buoyant force

M = metacenter

G = center of gravity of the body

B_O = center of buoyancy in upright position

B_O' = center of buoyancy in tilted position

MG = metacentric height or the distance from M to G

MB_O = distance from M to B_O

G_O = distance from G to B_O

v = volume of the wedge either immersion or emersion

s = horizontal distance between the center of gravity of the wedges

θ = angle of tilting

I = moment of inertia of the waterline section of the body

RM = righting moment

OM = overturning moment

Stability of Floating Bodies

Value of MB_O

$$MB_O = \frac{I}{V_D}$$

The formula above can be applied to any section.

For rectangular section

$$MB_O = \frac{B^2}{12D}\left(1 + \frac{\tan^2\theta}{2}\right)$$

Fluid Action on Surfaces

Problem Set 8

Problem 1

A plastic cube of side L and specific gravity of 0.82 is placed vertically in water. Is the cube stable?

Ans: Since $MB_O > GB_O$, M is above G. The body is stable.

Problem Set 8

Problem 2

A solid wood cylinder of specific gravity of 0.6 is 600 *mm* in diameter and 1200 *mm* high. If placed vertically in oil (s = 0.85), would it be stable?

Ans: Unstable (MB_O *(26.56 mm)* < GB_O *(176.5 mm)*)

Fluid Action on Surfaces

Problem Set 8

Problem 3

A rectangular scow 9 m wide, 15 m long, and 3.6 m high has a draft in sea water of 2.4 m. Its center of gravity is 2.7 m above the bottom of the scow. Determine the following:

3.1 The initial metacentric height.

3.2 The righting or overturning moment when the scow tilts until one side is just at the point of submergence.

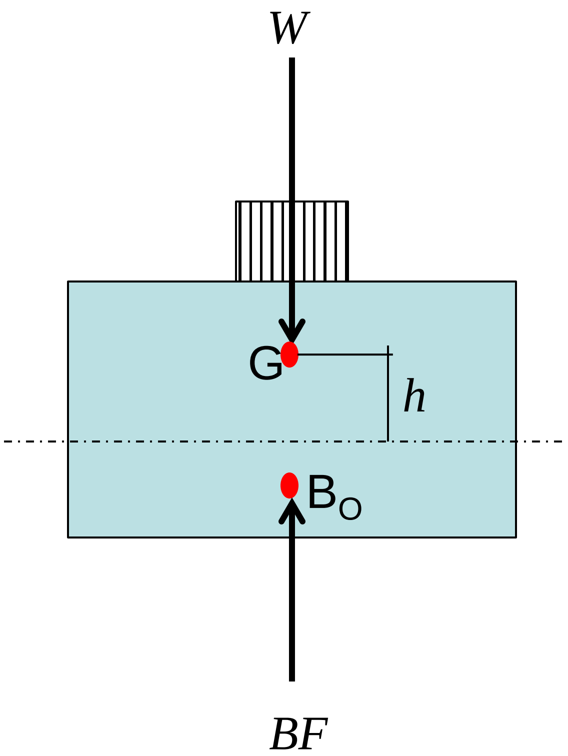
Ans: MG = 1.315 m, RM = 1,189.3 kN-m

Fluid Action on Surfaces

Problem Set 8

Problem 4

A wooden barge of rectangular cross-section is 8 *m* wide, 4 *m* high, and 16 *m* long. It is transporting in seawater (s = 1.03) a total load of 1,500 *kN* including its own weight and cargo. If a weight of 75 *kN* (included in the 1,500-*kN*) is shifted a distance of 2.5 *m* to one side, it will cause the barge to go down 450 *mm* in the wedge of immersion and also rise 450 *mm* in the corresponding wedge of emersion. The barge floats vertically before the shifting of the weight. Compute how far above the waterline is the center of gravity of the loaded barge.



Ans: h = 2.947 *m*